

SOME ASPECTS OF TRANSVERSITY*

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ABSTRACT

The specificities of transverse polarization with respect to helicity of ultrarelativistic fermions are pointed out. For massless fermions, a covariant transversity four-vector is defined, up to a kind of gauge transformation. The transversity distribution of quarks in a nucleon is defined. Its possible connection to the magnetic or electric dipole moment of the baryon is conjectured. Consequences of the approximate chiral invariance on transverse spin asymmetries in hard processes are enumerated. The "sheared jet effect" introduced by Collins for measuring the transverse polarization of a final quark is presented.

1 What is transversity ?

For a *massive* fermion, there is no problem of defining a transversely polarized particle of momentum \vec{p} : put it at rest, polarize it in some direction \hat{n} orthogonal to \vec{p} and then apply the necessary Lorentz boost to give it momentum \vec{p} . For a *massless* fermion, this definition does not work because there is no rest frame. On the other hand, we know that *helicity* states exist and form a complete basis. A transversely polarized state should therefore be a linear superposition of helicity ones. To get the coefficients, the most natural method is to give the particle a temporary small mass and let this mass go to zero. Thus, for \vec{p} along the positive z axis, we get

$$|\hat{n}\rangle = \frac{|+\rangle + e^{i\varphi}|-\rangle}{\sqrt{2}} \quad (1)$$

where φ is the azimuth of \hat{n} .

This recipe would not completely solve the problem in the case of a chiral symmetric world : the relative phase of the $|+\rangle$ and $|-\rangle$ states would be arbitrary, which would make the azimuth of \hat{n} ambiguous (a similar ambiguity would exist for linearly polarized photons if the electric-magnetic duality were an exact symmetry). Therefore the observability of transversity is linked to chiral symmetry breaking.

In the massive case, any polarization of the fermion can be represented in a noncovariant way by the 3-dimensional vector $\vec{P} = 2 \langle \vec{S} \rangle$ measured in the rest frame (i.e., before boost). Boosting \vec{P} results in the covariant polarization four-vector

$$\mathcal{S} = (0, \vec{P}_\perp) + P_L \frac{1}{m} \tilde{p} = \mathcal{S}_\perp + \mathcal{S}_L \quad (2)$$

where $\tilde{p} = (|\vec{p}|, p^0 \hat{p})$ and $\hat{p} \equiv \vec{p}/|\vec{p}|$.

When $m \rightarrow 0$, \mathcal{S}_L becomes infinite (unless P_L is strictly zero). However, the covariant projector $u(\vec{p}, s) \bar{u}(\vec{p}, s)$ keeps finite [1] :

$$u(\vec{p}, s) \bar{u}(\vec{p}, s) = \frac{1 + \gamma^5 \gamma \cdot \mathcal{S}}{2} (\gamma \cdot p + m) \rightarrow \frac{1 + \gamma^5 \gamma \cdot \mathcal{S}_\perp + P_L \gamma^5}{2} \gamma \cdot p \quad (3)$$

This equation shows that in the massless case helicity and transversity play very different roles. The later is associated with one more γ^μ matrix and is therefore called a *chirality odd* quantity [2]. Another interesting feature is the invariance under the "gauge" transformation of the transversity four-vector

$$\mathcal{S}_\perp^\mu \Rightarrow \mathcal{S}_\perp^\mu + \text{constant} \times p^\mu ; \quad (4)$$

this invariance makes the application of Lorentz transformations to \mathcal{S}_\perp^μ possible (it would not be the case if we had imposed $\mathcal{S}_\perp^0 \equiv 0$). The "gauge" $\mathcal{S}_\perp^\mu = (0, \vec{P}_\perp)$ is the analogue of the radiation gauge of the photon. One may view the gauge freedom as the relic of an infinitesimal uncertainty of the longitudinal polarization in the limit $m \rightarrow 0$.

2 Transversity distribution inside the nucleon

In analogy to the quark helicity distribution $\Delta_L q(x) \equiv 2g_1(x) = q^+(x) - q^-(x)$ in a longitudinally polarized nucleon N^+ , we define the quark transversity distribution [3, 4, 2] in a transversely polarized nucleon N^\uparrow :

$$\Delta_\perp q(x) \equiv 2h_1(x) = q^\uparrow(x) + q^\downarrow(x) \quad (5)$$

It obeys the Soffer's inequality [5]

$$|\Delta_\perp q(x)| \leq q^+(x) \quad (6)$$

which is stronger than the trivial one $|\Delta_\perp q(x)| \leq q(x)$.

$h_1(x)$ is not the same quantity as $g_1(x) + g_2(x)$, in spite of the fact that the later is measured with transversely polarized target. Only in a nonrelativistic quark model do they coincide (but such a model is unrealistic for deep inelastic reactions). In the infinite momentum frame,

$$\Delta_\perp q(x) = \frac{1}{4\pi} \int dz \langle N^\uparrow | \Psi_q^\dagger(0) (-\gamma^5 \vec{\gamma} \cdot \vec{P}_\perp) \Psi_q(0, 0, 0, z) | N^\uparrow \rangle e^{-ik_z z} \quad (7)$$

where $k_z = xp_z$ and $|N^\uparrow\rangle$ is a nucleon plane wave [normalized to $2p^0 (2\pi)^3 \delta(\vec{p} - \vec{p}')$] with transverse polarization \vec{P}_\perp . This quantity obeys the sum rule [2, 6]

$$\int_0^1 \Delta_\perp [q(x) - \bar{q}(x)] dx = \int \int \int d^3\vec{X} \langle N^\uparrow | \bar{\Psi}_q(\vec{X}) \vec{\Sigma} \cdot \vec{P}_\perp \Psi_q(\vec{X}) | N^\uparrow \rangle, \quad (8)$$

where $|N^\uparrow\rangle$ is now at rest and normalized to unity. The right-hand side of (8) is called the *tensor charge*. In fact, $\Sigma^i \equiv \epsilon_{ijk} \sigma^{jk}$ is part of the tensor $\sigma^{\mu\nu}$. This tensor occurs in the anomalous magnetic interaction $\frac{1}{2}\mu_a \bar{\Psi} \sigma^{\mu\nu} \Psi F_{\mu\nu}$, therefore we may conjecture that the anomalous magnetic moment of the quark contributes to the anomalous magnetic moment of the nucleon by (tensor charge) $\times \mu_a(\text{quark})$. Similarly for electric dipole moments. So, if we could tune these moments, one would get

$$\frac{\partial \mu(\text{nucleon})}{\partial \mu(\text{quark})} = \frac{\partial \text{e.d.m.}(\text{nucleon})}{\partial \text{e.d.m.}(\text{quark})} = \text{tensor charge} \quad (9)$$

3 Consequences of approximate chiral symmetry

Transverse spin asymmetries are interferences between different helicity amplitudes, the helicity difference being ± 1 . Two other equivalent statements are

- transverse spin asymmetries are helicity flip amplitudes of the unitarity diagram (*e.g.*, $\Delta_\perp q(x)$ is the helicity flip amplitude of forward $\bar{q} N \rightarrow \bar{q} N$ scattering).
- transverse spin asymmetries correspond to particle-antiparticle states of helicity ± 1 in the appropriate t-channels of the unitarity diagram ($N \bar{N} \rightarrow q \bar{q}$ for $\Delta_\perp q(x)$).

In deep inelastic reactions, quark masses can be neglected (if we discard charm and bottom quarks) and the axial or vector interactions with gauge bosons conserve helicity along the quark lines (chiral symmetry). Combined with the previous statement, this symmetry has the following consequences at leading twist (i.e. to zeroth order in m_q/Q or m_h/Q) [4]

- a) single spin asymmetries vanish
- b) there is no transversity correlation between external fermions not belonging to the same fermion line in the unitarity diagram : "transversity information is confined in quark lines".
- c) the polarized cross section is invariant if we rotate the \vec{P}_\perp 's of all the external fermions simultaneously by the same angle about their respective momenta (we may restrict the rotation to a subset of external fermions linked by fermion lines in the unitarity diagram).
- d) gluons do not contribute to the transverse spin of the nucleon
- e) the transverse spin correlation between target and projectile vanishes after integration over the azimuth of the final state.

Property a) is significantly violated for instance in $p + p \rightarrow \Lambda^\uparrow + X$ or $p + p^\uparrow \rightarrow \pi + X$, but at medium p_T ($\sim 1 - 2$ GeV) ; the problem may become acute if it persists at larger p_T . Property b) forbids measuring $\Delta_\perp q(x)$ using polarized beam and target in fully inclusive Deep Inelastic Lepton Scattering (DILS), contrary to $\Delta_L q(x)$. However, it allows $\Delta_\perp q(x)$ to be measured in doubly polarized Drell-Yan experiment [3] or in the semi-inclusive DILS [7, 8]

$$e^- N^\uparrow \rightarrow e^- \Lambda^\uparrow + X; \quad (10)$$

in the later case, one measures the polarization of the Λ which is proportional to $\Delta_\perp q(x) \times \Delta_\perp D_{\Lambda^\uparrow/q}(z)$. The second factor is the analysing power of the "quark polarimeter" $q^\uparrow \rightarrow \Lambda^\uparrow + X$.

Property *c*) gives for instance the $1 + A \cos[2\varphi_\mu - \varphi(\vec{P}_\perp) - \varphi(\vec{P}'_\perp)]$ dependence of doubly polarized Drell-Yan. It also imply property *e*) as well as the relations $A_{SS} = -A_{NN}$, $D_{SS} = D_{NN}$ between the double-spin asymmetry parameters.

4 The sheared jet effect

The fragmentation of a transversely polarized quark is *a priori* not invariant by rotation about the quark momentum [9] :

$$D_{h/q^\uparrow}(z, \vec{p}_\perp) = D_{h/q}(z, |\vec{p}_\perp|) \times \left\{ 1 + A_{h/q}(z, |\vec{p}_\perp|) |\vec{P}_\perp| \sin \left[\varphi(\vec{P}_\perp) - \varphi(\vec{p}_\perp) \right] \right\} \quad (11)$$

This effect might be a more efficient quark transversity polarimeter than $q^\uparrow \rightarrow \Lambda^\uparrow + X$. Both have to be calibrated at $e^+ e^-$ colliders [11, 10]. The single spin asymmetry observed in $p + p^\uparrow \rightarrow \pi + X$ may be a manifestation of it [12].

5 Conclusion

This very short introduction does not pretend to give an exhaustive view of the present developments of the physics of partonic transverse spin. It rather tries to bring out some particularities of this physics compared to that of partonic helicity, and to convince the reader that quite new information about hadron structure and chiral symmetry breaking may be obtained from its experimental study.

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